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The Hadronic Contribution to $(g - 2)_\mu$

Michel Davier^a

E-mail: davier@lal.in2p3.fr

^aLaboratoire de l'Accélérateur Linéaire, IN2P3/CNRS-Université de Paris-Sud,
BP34, 91898 Orsay, France

The evaluation of the hadronic contribution to the muon magnetic anomaly a_μ is reviewed, including a new estimate using precise results on the $\pi^+\pi^-$ spectral function from the KLOE Collaboration. It is found that the KLOE data confirm to some extent the previous e^+e^- annihilation data in this channel, and accentuate the disagreement with the isospin-breaking-corrected spectral function from $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays. Correcting for the empirical difference in the mass of the charged and the neutral ρ locally improves, but does not resolve this discrepancy. A preliminary reevaluation (including the KLOE data) of the e^+e^- -based Standard Model prediction of a_μ results in a deviation of 2.7 standard deviations from the BNL measurement.

1. Introduction

Hadronic vacuum polarization (HVP) in the photon propagator plays an important role in many precision tests of the Standard Model. This is the case for the muon anomalous magnetic moment $a_\mu \equiv (g_\mu - 2)/2$, where the HVP component is the leading contributor to the uncertainty of the Standard Model prediction. The HVP contribution is computed by means of a dispersion relation as an integral over experimentally determined spectral functions. It is the property of this dispersion relation that the $\pi\pi$ spectral function provides the major part of the total HVP contribution, so that the experimental effort focuses on this channel.

Spectral functions are directly obtained from the cross sections of e^+e^- annihilation into hadrons. The accuracy of the calculations has therefore followed the progress in the quality of the corresponding data[1]. Because the latter were not always suitable, it was deemed necessary to resort to other sources of information. One such possibility was the use of the vector spectral functions[2] derived from the study of hadronic τ decays[3] for the energy range less than $m_\tau \simeq 1.8 \text{ GeV}/c^2$. For this purpose, the isospin rotation that leads from the charged τ to the neutral e^+e^- final state has to be thoroughly corrected for isospin-breaking effects.

Also, it was demonstrated that essentially perturbative QCD could be applied to energy scales as low as 1–2 GeV[4,5], thus offering a way to replace poor e^+e^- data in some energy regions by a reliable and precise theoretical prescription[6–11].

Detailed reanalyses including all available experimental data have been published in Refs.[12–14] (see also the preliminary results given in Refs.[15,16]), taking advantage of precise results in the $\pi\pi$ channel from the CMD-2 experiment[17] and from the ALEPH analysis of τ decays[18], and benefiting from a more complete treatment of isospin-breaking corrections[19,20]. It was found that the e^+e^- and the isospin-breaking-corrected τ spectral functions do not agree within their respective uncertainties, thus leading to inconsistent predictions for the lowest-order hadronic contribution to a_μ . The dominant contribution to the discrepancy stems from the $\pi\pi$ channel with a difference of $(-11.9 \pm 6.4_{\text{exp}} \pm 2.4_{\text{rad}} \pm 2.6_{\text{SU}(2)} (\pm 7.3_{\text{total}})) \times 10^{-10}$, and a more significant energy-dependent deviation. When compared to the world average of the muon magnetic anomaly measurements, dominated by the results from the BNL experiment[21],

$$a_\mu = (11\,659\,208.0 \pm 5.8) \times 10^{-10}, \quad (1)$$

the respective e^+e^- and τ -based predictions disagreed at the level of 2.5 and 1.3 standard deviations, when adding experimental and theoretical

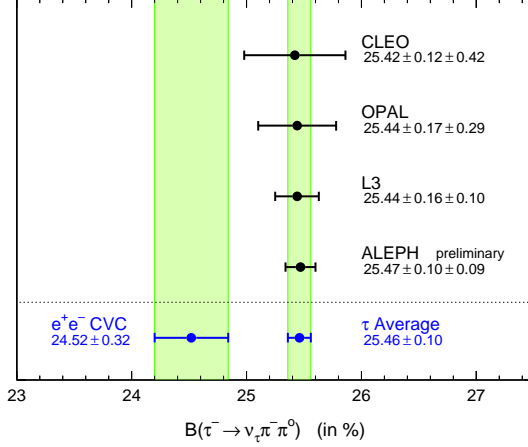


Figure 1. *The measured branching ratios for $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ compared to the prediction from the $e^+e^- \rightarrow \pi^+\pi^-$ spectral function applying the isospin-breaking correction factors discussed in Ref. [13]. The measured branching ratios are from ALEPH [18], CLEO [22] and OPAL [23]. The L3 and OPAL results are obtained from their $h\pi^0$ branching ratio, reduced by the small $K\pi^0$ contribution measured by ALEPH [24] and CLEO [25].*

errors in quadrature.

The problem between τ and e^+e^- data is more noticeable when comparing the $\tau \rightarrow \pi\pi^0\nu$ branching fraction with the prediction obtained from integrating the corresponding isospin-breaking-corrected e^+e^- spectral function. Here, the function under the integrand is less selective than it is the case for the HVP contribution to a_μ , leading to a discrepancy of 2.9 standard deviations[12], as shown in Fig. 1.

This summer, new data on the $\pi\pi$ spectral function in the mass region between 0.60 and 0.97 GeV/ c^2 were presented by the KLOE Collaboration[26], using the—for the purpose of precision measurements—innovative technique of the radiative return[27]. The statistical precision of these data by far outperforms the Novosibirsk sample, but the systematic errors are about twice as large as those obtained by CMD-2. New data

using the same technique have been published by the BABAR Collaboration[28] on the $\pi^+\pi^-\pi^0$ final state. They unveil a larger cross sections and a resonant peak at around 1.6 GeV/ c^2 that was missed by the previous DM2 measurement[29]. The BABAR data are not (yet) used in the preliminary reevaluation of the lowest-order HVP contribution given here. The correction to $a_\mu^{\text{had,LO}}$ when using the BABAR data for this mode [30] is of the order of $+1 \times 10^{-10}$. Also, preliminary BABAR results are available on the $2\pi^+2\pi^-$ final state which are overall more precise than existing data [30]. They are not included in the present evaluation and their effect would be to change the hadronic contribution by about -1×10^{-10} .

2. Muon Magnetic Anomaly

It is convenient to separate the Standard Model (SM) prediction for the anomalous magnetic moment of the muon into its different contributions,

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{weak}}, \quad (2)$$

with

$$a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,HO}} + a_\mu^{\text{had,LBL}}, \quad (3)$$

and where $a_\mu^{\text{QED}} = (11\,658\,472.0 \pm 0.2) \times 10^{-10}$ is the pure electromagnetic contribution (see[31,32] and references therein), $a_\mu^{\text{had,LO}}$ is the lowest-order HVP contribution, $a_\mu^{\text{had,HO}} = (-10.0 \pm 0.6) \times 10^{-10}$ is the corresponding higher-order part[33,2], and $a_\mu^{\text{weak}} = (15.4 \pm 0.1 \pm 0.2) \times 10^{-10}$, where the first error is the hadronic uncertainty and the second is due to the Higgs mass range, accounts for corrections due to exchange of the weakly interacting bosons up to two loops[34]. For the light-by-light (LBL) scattering part, $a_\mu^{\text{had,LBL}}$, we use the value $(12.0 \pm 3.5) \times 10^{-10}$ from the latest evaluation[35], slightly corrected for the missing contribution from (mainly) the pion box.

Owing to unitarity and to the analyticity of the vacuum-polarization function, the lowest order HVP contribution to a_μ can be computed via the dispersion integral[36]

$$a_\mu^{\text{had,LO}} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R^{(0)}(s), \quad (4)$$

where $K(s)$ is a well-known QED kernel, and $R^{(0)}(s)$ denotes the ratio of the “bare” cross section for e^+e^- annihilation into hadrons to the pointlike muon-pair cross section. The bare cross section is defined as the measured cross section corrected for initial-state radiation, electron-vertex loop contributions and vacuum-polarization effects in the photon propagator. However, photon radiation in the final state is included in the bare cross section defined here. The reason for using the bare (*i.e.*, lowest order) cross section is that a full treatment of higher orders is anyhow needed at the level of a_μ , so that the use of the “dressed” cross section would entail the risk of double-counting some of the higher-order contributions.

The function $K(s) \sim 1/s$ in Eq. (4) gives a strong weight to the low-energy part of the integral. About 91% of the total contribution to $a_\mu^{\text{had,LO}}$ is accumulated at center-of-mass energies \sqrt{s} below 1.8 GeV and 73% of $a_\mu^{\text{had,LO}}$ is covered by the $\pi\pi$ final state, which is dominated by the $\rho(770)$ resonance.

3. The Input Data

A detailed compilation of all the experimental data used in the evaluation of the dispersion integral (4) is provided in Refs.[13,12]. Also discussed therein is the corrective treatment of radiative effects applied to some of the measurements. The τ spectral function is obtained by averaging the results from ALEPH[3], CLEO[37] and OPAL[38], which exhibit satisfactory mutual agreement.

A comparison of the $e^+e^- \rightarrow \pi^+\pi^-$ data and the corresponding τ spectral function, represented as a point-by-point ratio to the τ spectral function is given in Fig. 2. Several observations can be made.

- A significant discrepancy, mainly above the ρ peak is found between τ and the e^+e^- data from CMD-2 as well as older data from OLYA.
- Overall, the KLOE data confirm the trend exhibited by the other e^+e^- data.
- Some disagreement between KLOE and CMD-2 occurs on the low mass side (KLOE

data are large), on the ρ peak (KLOE below CMD-2) as well as on the high mass side (KLOE data are low).

At this stage, the τ spectral function has not been corrected for a possible $\rho^- - \rho^0$ mass and width splitting[41,40]. In contrast to earlier experimental[3] and theoretical results[39], a combined pion form factor fit[40] to the new precise data on e^+e^- and τ spectral functions leads to $m_{\rho^-} - m_{\rho^0} = (2.3 \pm 0.8) \text{ MeV}/c^2$, while no significant width splitting is observed within the fit error of $1.7 \text{ MeV}/c^2$.

Note that if the mass difference is to be taken as an experimental fact, a larger width difference would be expected. Using a chiral model of the ρ resonance[42,19,20], one has

$$\Gamma_{\rho^0} = \Gamma_{\rho^-} \left(\frac{m_{\rho^0}}{m_{\rho^-}} \right)^3 \left(\frac{\beta_0}{\beta_-} \right)^3 + \Delta\Gamma_{\text{EM}} \quad (5)$$

where $\Delta\Gamma_{\text{EM}}$ is the width difference from electromagnetic decays. This leads to a total width difference of $(2.1 \pm 0.5) \text{ MeV}/c^2$ that is marginally consistent with the observed value[40]. is observed within the fit error of $1.7 \text{ MeV}/c^2$.

Considering the mass splitting in the isospin-breaking correction of the τ spectral function tends to locally improve (though not restore) the agreement between τ and CMD-2 data, leaving an overall normalization discrepancy. Increasing the $\Gamma_{\rho^-} - \Gamma_{\rho^0}$ width splitting by $+3 \text{ MeV}/c^2$ improves the agreement between τ and KLOE data in the peak region, while it cannot correct the discrepancies in the tails. Note that a correction of the mass splitting alone would *increase* the discrepancy between the τ and e^+e^- -based results for $a_\mu^{\text{had,LO}}$.

During the previous evaluations of $a_\mu^{\text{had,LO}}$, the results using respectively the τ and e^+e^- data were quoted individually, but on the same footing since the e^+e^- -based evaluation was dominated by the data from a single experiment (CMD-2). The confirmation of this discrepancy by KLOE discredits the τ -based result for the use in the dispersion integral until a better understanding of the dynamical origin of the observed effect is achieved. This is a challenging problem, which may itself turn out to be of fundamental impor-

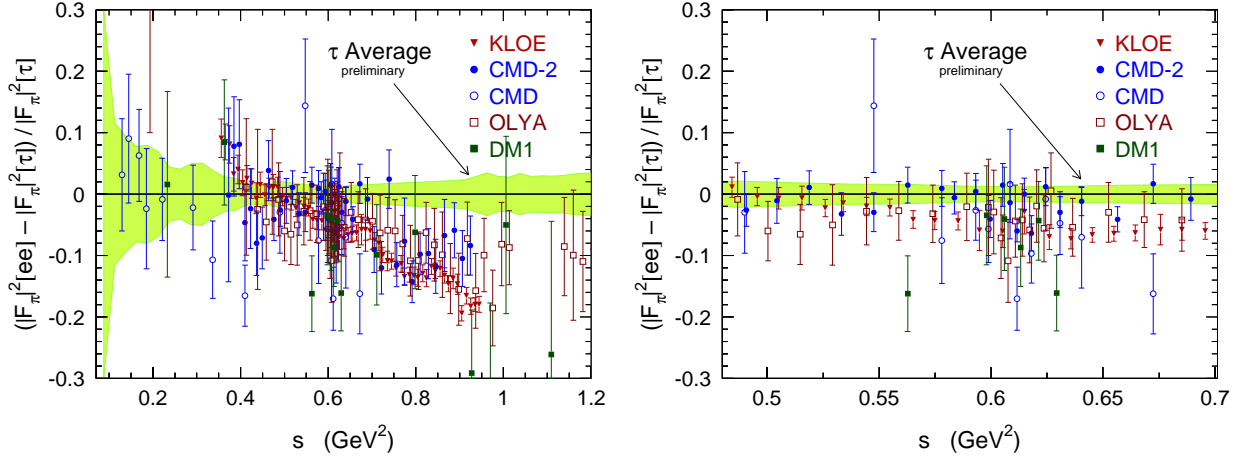


Figure 2. Relative comparison of the $\pi^+\pi^-$ spectral functions from e^+e^- -annihilation data and isospin-breaking-corrected τ data, expressed as a ratio to the τ spectral function. The shaded band indicates the errors of the τ data. The e^+e^- data are from KLOE[26], CMD-2[17], CMD, OLYA and DM1 (references given in Ref.[12]). The right hand plot emphasizes the region of the ρ peak.

tance.

4. Results

The inclusion of the KLOE $\pi\pi$ data decreases the contribution from this mode from [12] $(450.2 \pm 4.9 \pm 1.6_{\text{rad}}) \times 10^{-10}$ to $(448.3 \pm 4.1 \pm 1.6_{\text{rad}}) \times 10^{-10}$ for the energy interval between 0.5 and 1.8 GeV. Note that the additional systematic error due to radiative effects originates from the energy regions not covered by the recent KLOE and CMD-2 measurements, where a full treatment of radiative corrections is applied. The preliminary estimate of the integral (4) given below includes one additional improvement with respect to Ref.[12]: perturbative QCD is used instead of experimental data in the region between 1.8 and 3.7 GeV, where non-perturbative contributions to integrals over differently weighed spectral functions were found to be small[7]. This results in a reduction of $a_\mu^{\text{had,LO}}$ by -1×10^{-10} . All other contributions to the dispersion integral are equal to those defined in Ref.[12].

The e^+e^- -based result for the lowest order

hadronic contribution is

$$a_\mu^{\text{had,LO}} = (693.4 \pm 5.3 \pm 3.5_{\text{rad}}) \times 10^{-10}, \quad (6)$$

where the second error is due to our treatment of (potentially) missing radiative corrections in the older data[13]. Adding to this the QED, higher-order hadronic, light-by-light scattering, and weak contributions given in Section 2, one finds

$$a_\mu^{\text{SM}} = (11\,659\,182.8 \pm 6.3_{\text{had,LO+HO}} \pm 3.5_{\text{had,LBL}} \pm 0.3_{\text{QED+EW}}) \times 10^{-10}. \quad (7)$$

This value can be compared to the present measurement (1); adding all errors in quadrature, the difference between experiment and theory is

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25.2 \pm 9.2) \times 10^{-10}, \quad (8)$$

which corresponds to 2.7 “standard deviations” (to be interpreted with care due to the dominance of systematic errors in the SM prediction). A graphical comparison of the result (7) with previous evaluations and the experimental value is given in Fig. 3.

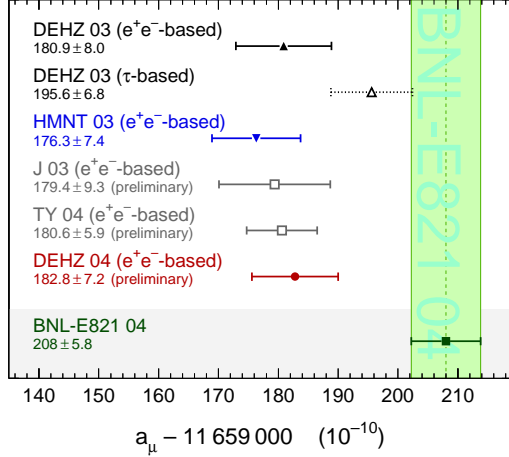


Figure 3. Comparison of the result (7) with the BNL measurement[21]. Also given are our previous estimates[12], where the triangle with the dotted error bar indicates the τ -based result, as well as the estimates from Refs.[14-16], not yet including the KLOE data.

5. Conclusion and Perspective

In spite of the new and precise data on the two-pion spectral function from the KLOE Collaboration, the lowest order hadronic vacuum-polarization contribution remains the most critical component in the Standard Model prediction of a_μ . The central piece of information provided by the present KLOE data is that they confirm the discrepancy between the τ data and e^+e^- annihilation observed in this channel[12]. This said, we point out that there also occurs disagreement between KLOE and CMD-2 data in some of the energy regions.

An empirical isospin-breaking correction of the ρ resonance lineshape (mass and width) improves but does not restore the agreement between the two data sets. It is a consequence of this confirmation that, until the CVC puzzle is solved, only e^+e^- data should be used for the evaluation of the dispersion integral. Doing so, and including the KLOE data, we find that the Standard Model prediction of a_μ differs from the experi-

mental value by 2.7 standard deviations.

We are looking forward to the forthcoming results on the low- and high-energy two-pion spectral function from the CMD-2 Collaboration. These data will help to significantly reduce the systematic uncertainty due to the corrective treatment of radiative effects, often omitted by part by the previous experiments.

The initial-state-radiation program of the BABAR collaboration has already proved its performance by publishing the spectral function for $\pi^+\pi^-\pi^0$ (and soon for $2\pi^+2\pi^-$), while results for the two-pion final state are expected.

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